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Stress space: striated faults, deformation twins, and their constraints on paleostress

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Abstract

Stress space is used to illustrate a discussion of the range of stress tensors theoretically compatible with striated faults, and with deformation twinning. Other than friction laws, each of the main constraints placed on stress by such data specifies the magnitude of a component of traction. Being linear in elements of the stress tensor, such a constraint by each datum is represented in stress space by a hyperplane. Comparison at each level of constraint (orientation, sense of shear, assumption of critical ratio or value of shear stress for faulting/twinning) is made for a single datum, followed by consideration of sets of data, and stress tensor reduction. At no level of constraint are striated faults and deformation twins equivalent. Fault data cannot constrain absolute values of stress components. Data for untwinned crystals within twinned/untwinned sets provide the fullest limit to values of deviatoric stresses. Paleostress estimates from twinned/untwinned data are generally preferable to those from striated faults. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

1.1. Background

Over the last three decades, many attempts have been made to deduce the stress state of part of the Earth in the geological past ("paleostress"). Although it is wise to incorporate evidence from as wide a range of phenomena as possible (e.g. Lisle and Vandycke, 1996), many published studies are restricted to a single type of data. One popular approach is the analysis of fault striae ("slickenlines"), following the work of Carey and Brunier (1974), Angelier (1975), Armijo and Cisternas (1978), Angelier and Goguel (1979) and Etchecopar et al. (1981). Another is the analysis of crystal orientations of twinned and untwinned mineral grains in thin section, following among others Turner (1953, 1962), Nissen (1964), Spang (1972) and Jamison and Spang (1976).

A constant problem for such paleostress analyses is the difficulty of separating phases of a polyphase stress history. Nemcok et al. (1999) propose a method of analysis which facilitates identification of clusters of like data, and determines an appropriate stress state for each cluster. In doing so, they treat fault striation data and deformation twin data as equivalent, despite mentioning that they offer different constraints. This contrasts with previous work, particularly of Laurent and co-workers (e.g. Laurent et al., 1981; Lacombe et al., 1990, 1992; Lacombe and Laurent, 1992), who take full account of the differences between the two types of data, and whose recent calibrations using natural samples, numerical simulations (Laurent et al., 1990) and experimentally deformed samples (Lacombe and Laurent, 1996) justify more confidence in stress estimation using analyses of calcite twinning than that using striated faults.

1.2. Scope and starting point of this paper

This paper is a contribution to discussion. It takes the form of a comparison of the constraints placed by striated faults and by deformation twins on an assumed common generating stress state. The reasonableness, or otherwise, of the assumption that real structures result from a common stress state will be mentioned in Section 6, but this is not the main issue addressed in this paper. The problematic nature of cluster analysis as part of these procedures (Nemcok et al., 1999) will be discussed elsewhere.

All the discussion below relies on the tensorial relationship of traction, $\mathbf{t}$, to fault or twin plane orientation, here given algebraic form $\mathbf{t} = \boldsymbol{\sigma}.\mathbf{n}$, where $\mathbf{n}$ is unit normal to the fault or twin plane and $\boldsymbol{\sigma}$ is the stress tensor. Apart from differences of terminology and symbolism, the essence of the theory of stress, as set out by such standard works as Nye (1957) and Jaeger and Cook (1969, 1979), is consistent in all the main literature and well presented by Malvern (1969). Traction is the term used here to mean the limit, as area tends to zero, of force per unit area of a plane, on one side of

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the plane. Its in-plane and normal components (commonly "*shear stress*" and "*normal stress*") are adequate for considering an isotropic fault plane. For deformation twinning, it is necessary to consider its component along an in-plane linear direction—that of the crystallographically controlled displacement corresponding to initiation or growth of deformation twin.

1.3. Terms, symbols and sign conventions

Several competing terminological conventions have long been established in the literature. Most authors use, for **t** above, some phrase that includes either the word "stress" or the word "traction". For example, the student text "The Earth" (Verhoogen et al., 1970, p.467) reports that it "… is termed the *stress vector* or *surface traction*…". The core of this paper is discussion of vectorial representation of tensor $\boldsymbol{\sigma}$, not **t**. So, to avoid confusion, the term "stress vector" and the symbol "**s**" will not be used for either in this paper. The term "*σ-vector*" will be used for a vectorial representation of the stress state (three-dimensional in real space, conceptualised as tensor $\boldsymbol{\sigma}$), while "*traction*" (hence symbol **t**) is used for its expression on a two-dimensional plane in real space.

Different conventions exist also with regard to symbols and signs. This paper extends the ideas of Fry (1999) and therefore follows the same usage. Symbols for vectors and tensors are in bold, while the elements of the matrix representation of a tensor are normal weight followed by row and column subscripts. To avoid any confusion with strain (as well as traction), stress is given tensor symbol $\boldsymbol{\sigma}$ not **s**.

For conceptual simplicity, the sign convention of general physics is adhered to. All vectors representing parameters at a point are positive outward, and all vectors of same polarity have same sign. So, positive traction is in the sense ("tension") which leads to positive displacement and positive strain ("extension") and in the same sense (outward) as a positive vector normal to that side of a plane to which it lies. These conventions, which prevent a great deal of confusion when relating stress, strain and displacement, conflict at one point, as detailed in the text, with previous usage deriving from the peculiar geological convention that compression is positive.

1.4. Stress space representation

To assist appreciation, mathematical relationships are given geometrical representation. The space used—the *unreduced* "stress space" of yield surface and plastic potential surface analysis (e.g. Hobbs *et al.*, 1990), called "*σ-space*" by Fry (1999) and below—is six dimensional and has orthogonal axes for the values of the six independent matrix elements σ_{11}, σ_{22}, σ_{33}, σ_{23}, σ_{31}, σ_{12} of the stress tensor.

The methods discussed will only limit estimates of deviatoric stress, corresponding geometrically to intersections of less than six dimensions in σ-space. Such tensor reduction is not identical for all data types and all working assumptions, and will be considered in Section 5.

It is convenient sometimes to consider the stress tensor to be represented in σ-space by the *point* (σ_{11},σ_{22},σ_{33},σ_{23},σ_{31},σ_{12}), and sometimes by the "*σ-vector*", which extends to this point from the origin (0,0,0,0,0,0). Geometrical features in σ-space are illustrated here by two- and three-dimensional analogues.

The strength of σ-space representation for paleostress rests on the fact that $L = \mathbf{v}.\boldsymbol{\sigma}.\mathbf{n}$—the dot product of the traction vector with a real space vector, **v**—is linear in σ_{ij}. This is clearly seen in the expanded form

$$L = v_1n_1\sigma_{11} + v_2n_2\sigma_{22} + v_3n_3\sigma_{33} + (v_2n_3 + v_3n_2)\sigma_{23} + (v_3n_1 + v_1n_3)\sigma_{31} + (v_1n_2 + v_2n_1)\sigma_{12}. \quad (1)$$

If **v** is a unit vector specifying any general direction, L is the magnitude of the component of traction along it. Specifying a value of L limits the locus in σ-space of all stress states with which it is compatible to a hyperplane. The term "hyperplane" is used here to indicate that the locus extends infinitely in one less dimension than the space in which it resides. As such, it has a normal in the remaining σ-space dimension. Equality $L = 0$ specifies a five-dimensional hyperplane through the origin (Fry, 1999). It divides σ-space into the two half-spaces $L > 0$ and $L < 0$. Equality $L = k$ defines a hyperplane parallel to $L = 0$, but displaced from the σ-space origin by a distance dependent on k.

2. One striated fault-levels of constraint

2.1. Orientation

The theoretical basis of the use of a striated fault to determine stress state is the combination of two assumptions. The first is the validity of the generally accepted theory of stress, encapsulated by the tensorial relationship $\mathbf{t} = \boldsymbol{\sigma}.\mathbf{n}$, mentioned above. The second is that the striation on the fault indicates the direction of the in-plane component of the traction vector along the fault plane (the "resolved shear stress"). The latter condition may be restated as

$$\mathbf{b}.\boldsymbol{\sigma}.\mathbf{n} = 0 \quad (2)$$

where **b** is a unit vector in the plane of the fault, at right angles to the striation. This constrains the σ-vector to lie somewhere in the hyperplane of Eq. (2) in σ-space (Fry, 1999). The three-dimensional analogue of this is shown in Fig. 1(a).

2.2. Shear sense

Shear sense is a subsidiary constraint. It does not of itself constrain the stress tensor but, when added to the fault plane and striation orientations, it further restricts the permitted range. The requirement that the component of traction have

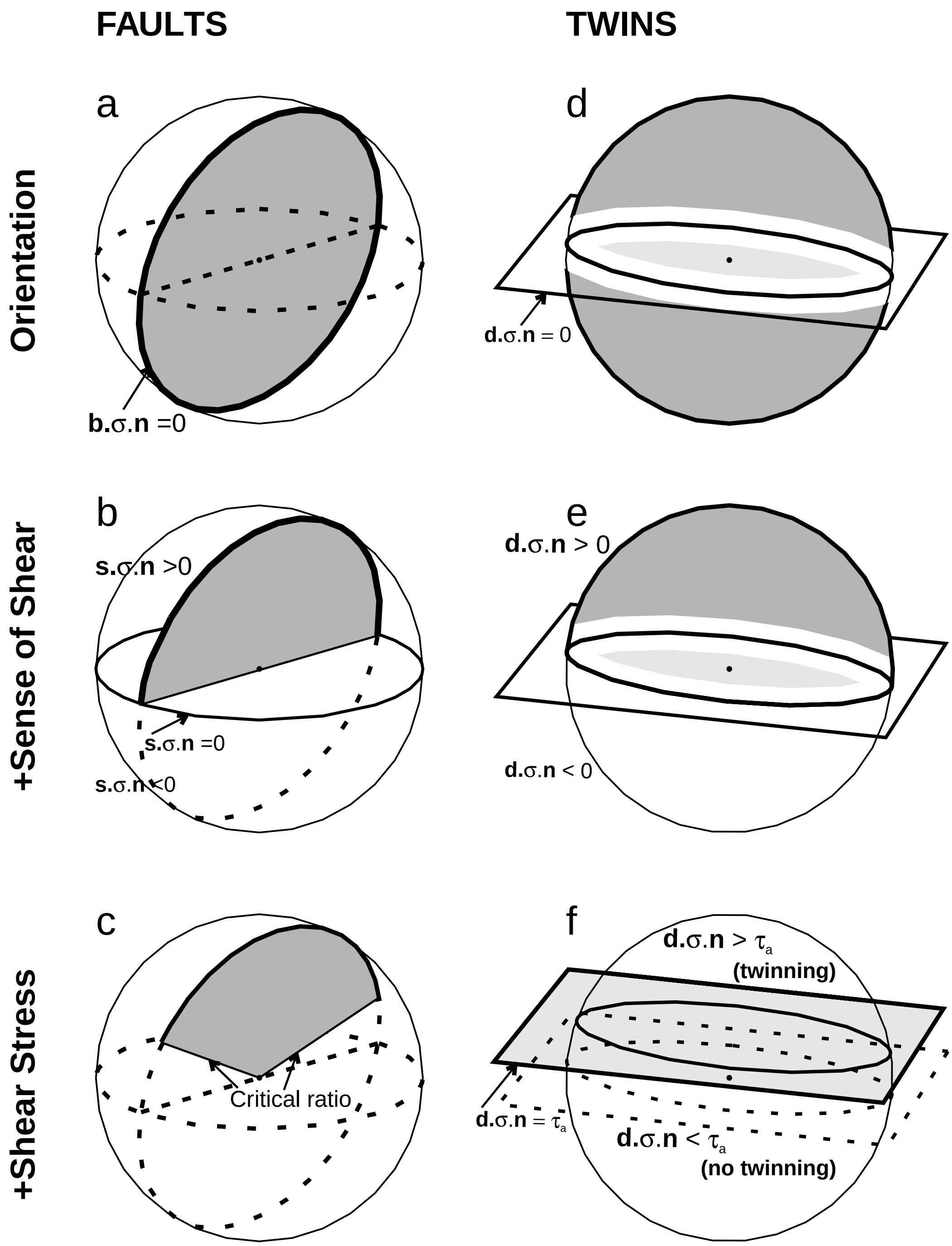

Fig. 1. Three-dimensional analogues of σ-space for a single datum of a striated fault (a–c) and a deformation twin (d–f), for known orientations only (a, d), orientations and shear sense (b, e) and with an assumed minimum ratio (c) or value (f) of shear stress component. The bounding spheres and great circles are only aids to visualisation, by their intersection, of σ-space (hyper)planes and (hyper)volumes which extend without limit away from the origin. Dark shading in (a)–(e) identifies the hyperplane or hypervolume which is the locus of possible points representing the stress tensor. The significance of the lightly shaded $\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} = 0$ or $\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} > 0$ hyperplane is different in (d), (e) and (f), as given below. (a) Fault plane and striation orientations limit the σ-vector locus to the hyperplane $\mathbf{b}.\boldsymbol{\sigma}.\mathbf{n} = 0$. (b) The locus in (a) is reduced to the half within half-space $\mathbf{s}.\boldsymbol{\sigma}.\mathbf{n} > 0$ if shear sense is known. (c) According to how a friction law is applied, (i) assumption of failure *at any value above* the critical threshold ratio of shear to normal stress reduces the extent of the stress locus from that in (b), but not its dimensionality, whereas (ii) assumption of failure *at the threshold* reduces the dimensionality of the locus by one. (d) Deformation twinning could, in terms of orientation alone, be consistent with any stress state not on the lightly shaded $\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} = 0$ hyperplane of zero component of shear stress. (e) The sense of shear brought about by twinning limits the stress state to the half space $\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} > 0$, to one side of the lightly shaded $\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} = 0$ hyperplane. (f) A critical shear stress for twinning, τ_a, divides σ-space into two parts—one consistent with twinning, the other consistent with no twinning. These are separated by an off-centre hyperplane boundary $\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} = \tau_a$ (light shading). Therefore, distance as well as direction from the σ-space origin is constrained by any twinned or untwinned datum.

correct sense (in the same sense as the displacement direction, specified by unit vector **s**) can be given as the condition $\mathbf{s}.\boldsymbol{\sigma}.\mathbf{n} > 0$. Only that half of hyperplane $\mathbf{b}.\boldsymbol{\sigma}.\mathbf{n} = 0$ lying in the half-space $\mathbf{s}.\boldsymbol{\sigma}.\mathbf{n} > 0$ is permitted by both orientations and shear sense, analogous to Fig. 1(b).

2.3. Friction

Adding frictional constraint to the determination of a stress tensor is complex (e.g. Célérier, 1988), and is not easily represented in three or fewer dimensions. However, some aspects of its effect on the locus of possible σ-vectors in σ-space can be appreciated qualitatively. In Fig. 1(b) the σ-vector locus is bounded by the intersection of $\mathbf{b}.\boldsymbol{\sigma}.\mathbf{n} = 0$ (orientation) and $\mathbf{s}.\boldsymbol{\sigma}.\mathbf{n} = 0$ (zero shear stress). The intersection of hyperplanes in full σ-space, to which that in Fig. 1(b) is analogous, is four-dimensional. Searching for the σ-space locus representing a non-zero critical ratio of shear to normal stress corresponds to moving and bending this four-dimensional hypersurface through the five dimensions of $\mathbf{b}.\boldsymbol{\sigma}.\mathbf{n} = 0$, away from the restricted locus of conditions of zero shear stress ($\mathbf{s}.\boldsymbol{\sigma}.\mathbf{n} = 0$), into conditions with shear stress of the correct sense ($\mathbf{s}.\boldsymbol{\sigma}.\mathbf{n} > 0$). A number of points can be highlighted using the analogue, Fig. 1(c).

1. Conditions compatible with a friction law are more restricted than those permitted by only orientation and sense of shear.
2. If angle of friction is assumed to be independent of confining stress, failure can be reproduced at any multiple of a critical set of stress values. Consequently, critical conditions have a locus in σ-space made up of radii from its origin.
3. Conditions *at* a critical threshold have one dimension less in σ-space than conditions *bounded by* such a threshold.
4. There is no general incompatibility between requirements of the frictional constraint and those of the orientational and shear sense constraints.

3. One twinnable crystallographic plane-levels of constraint

3.1. Orientation

The direction of displacement that may be brought about by deformation twinning is dictated by crystal orientation. If the permitted direction is specified by a unit vector **d** (**m** of Laurent et al., 1981), the component of the traction in the displacement direction is given by $\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n}$. Orientation alone provides only the constraint of prohibiting twinning when

$$\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} = 0, \tag{3}$$

analogous to Fig. 1(d).

3.2. Shear sense

When the sense of displacement by twinning is taken into account, the condition for twinning becomes $\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} > 0$. This defines the six-dimensional half space to one side of the Eq. (3) hyperplane, Fig. 1(e). This half space contains the six half-axes, σ_{ij}, of same sign as their coefficients in the expanded form of Eq. (3). (To be consistent with the rest of this paper, positive **d** is here in the sense of positive traction causing positive displacement of the positive **n** side relative to the negative **n** side of the twin plane. This is the opposite sign convention to that used by Laurent et al. (1990).)

3.3. Yield stress

Initiation or growth of a deformation twin takes place at a yield shear stress, typically designated τ_a, which is theoretically constant and which in calcite has been experimentally confirmed to be independent of temperature, confining (normal) stress, strain and strain rate, provided temperature is above a minimum threshold value for activation. Conditions inducing twinning ($\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} > \tau_a$) and those which do not ($\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} < \tau_a$) lie each side of the critical condition

$$\mathbf{d}.\boldsymbol{\sigma}.\mathbf{n} = \tau_a. \tag{4}$$

The offset of the hyperplane of Eq. (4) from the σ-space origin, analogous to that in Fig. 1(f), is determined by the yield stress of the particular twin system used in the analysis.

The above statements concern a single crystal. The relationship between boundary stresses and twinning in real polycrystalline samples is discussed briefly in Section 6.3.

4. Stress tensor estimation from sets of data

4.1. Striated fault sets—orientation and shear sense

The illustrations in Fig. 1, and their consideration above, refer to the constraint on the stress state by a single fault or twin datum. The constraint given by a group of fault data is illustrated in similar fashion in Fig. 2(a). This set of three data is analogous to a set of more than four data in σ-space. One datum has been included of each level of constraint detailed above and in Fig. 1. This set illustrates two important observations.

1. Use of some data of known shear sense does not preclude the inclusion of other data for which shear sense is unknown, as emphasised by Fry (1999).
2. Optimisation in terms of orientational constraint can be accomplished by minimising a function of angular errors in σ-space for the hyperplanes (Fig. 2a).

Of possible functions for the latter, the sum of squares of sines of angles between a stress estimate and the hyperplanes is computationally simple to minimise. The

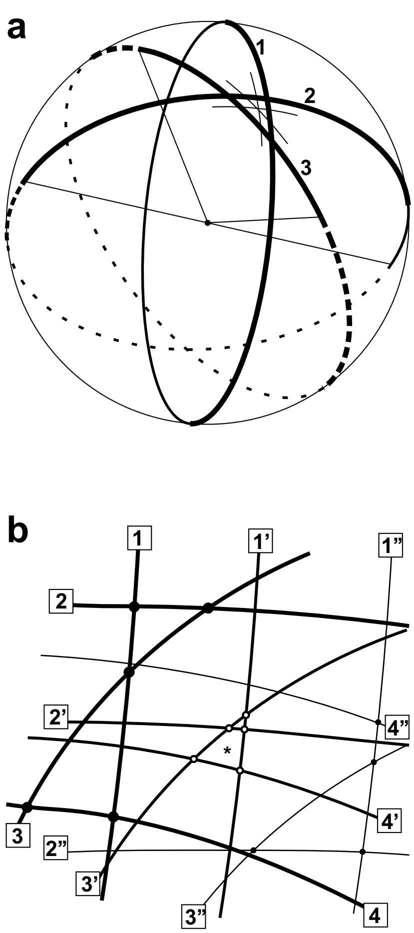

Fig. 2. Analogues of σ-space for fault data sets. (a) A three-dimensional analogue, with complete or partial hyperplanes for three data represented as unbroken lines on a spherical surface, as in Fig. 1(a–c). (Datum 1: orientation only. Datum 2: orientation and sense of shear. Datum 3: with additional assumption about friction.) Their intersection would give σ-space co-ordinates representing the stress tensor capable of generating all three data. They do not intersect perfectly. Optimal allowance of a small error on each (thin lines) permits their intersection, at a location representing a best estimate of common generating stress. (b) Cartoon two-dimensional analogue of part of σ-space to illustrate how assumption of different values of frictional parameter(s) (usually angle of friction) gives different loci for failure. Loci for three values ($n'' > n' > n$) are shown for each of four numbered faults ($n = 1,2,3,4$): heavy lines (1,2,3,4; heavy dot intersections); medium lines (1',2',3',4'; open circle intersections); fine lines (1'',2'',3'',4''; small dot intersections). The medium weight lines come closest to having a common intersection (asterisk). Their condition is the best estimate of the frictional parameter, and their intersection indicates the candidate stress state.

conceptualisation offered by Fry (1999), of minimising the second moment of the set of unit vector normals to the hyperplanes, is mathematically identical. A more robust algorithm could also be formulated. Whatever algorithm is adopted, it should accommodate instances where fault data permit more than one degree of freedom in the stress tensor estimate (Fry, 1999). Geometrically, this corresponds to the fact that a hyperplane intersection in σ-space, analogous to that shown in Fig. 2(a), may have more than one dimension.

4.2. Striated fault sets-including friction

The following observations are offered regarding how friction should be taken into account when evaluating a best fit of stress tensor to fault data.

1. Orientational accuracy, shear sense correctness and frictional consistency are not independent constraints. Any estimated frictional condition subsumes shear sense, and a frictional function is concurrently a function of orientation. So, these constraints do not provide alternative stress values, such as would be represented as alternative loci in σ-space or Fig. 2(a). Instead, they are nested, the frictionally acceptable locus lying as a subset within the shear sense acceptable locus within the locus acceptable in terms of orientations alone. Therefore, it is not appropriate to evaluate a best-fit stress tensor by summing separate relative weightings allocated to orientational accuracy, to shear sense correctness and to frictional consistency. In general, this nesting of constraints favours a stepwise determination—1 orientation; 2 shear sense; 3 friction.
2. Frictional optimisation is for mutual consistency (Fig. 2b), in contrast to orientational optimisation for accuracy of match to data. Also, such frictional optimisation assumes constancy throughout the space and time of the data of an additional, possibly more variable, stress parameter—the ratio of deviatoric stress to effective confining stress—implying an approximately constant ratio of pore fluid pressure to confining pressure. Further, real frictional behaviour may be more complex, involving a non-linear friction law with cohesion. This will result in critical conditions, which are not radial from the origin in σ-space, making their geometric appreciation even more problematic.

In the light of both these observations, it would seem wise to find out first if the orientation and shear sense data are compatible with a common stress state, before considering frictional consistency. Whatever law may then be assumed for the latter, different best estimates of frictional parameter and stress tensor may be obtained, corresponding to different best locations in Fig. 2(b). These will depend whether the function for fitting to the friction law involves only error terms for frictional parameters, or regression involving a trade-off between error terms in the orientational and frictional parameters.

4.3. Deformation twin sets-orientation and shear sense

Fig. 3 shows cartoon two-dimensional analogues of a portion of σ-space about its origin. For comparison with twinned/untwinned data, striated fault data are shown in Fig. 3(a). They are represented by geometric elements, which radiate linearly without limit from the origin, and therefore they would be better visualised as their intersections with a locus of unit radius (circle/sphere/hypersphere), as in Fig. 1 and Fig. 2(a).

As with a single datum, orientation and sense of displacement give less constraint for twinning (Fig. 3b) than for faulting (Fig. 3a). With regard to untwinned crystal planes, lack of twinning does not necessarily imply incorrect orientation or sense; the component of shear stress in the displacement direction may be less than critical. Therefore, the sense of untwinned data cannot be used. With regard to the twinned planes, each datum limits the range of possible directions of σ-vector from the origin (Fig. 3b). It might be thought that, with a large enough number of twinned data, there would be sufficient critically limiting angles in σ-space to give good constraint on the σ-vector. However, as can be shown by consideration of Fig. 3(c), if the σ-vector is small relative to τ_a, the σ-space angle between limiting twinned data will always be large, corresponding to poor constraint on deviatoric stress.

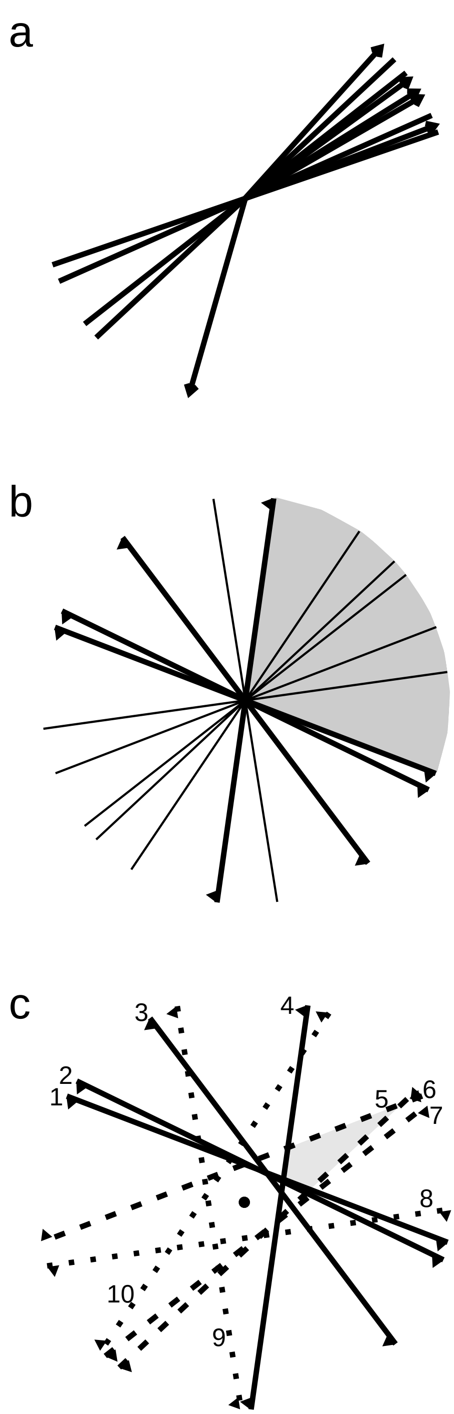

Fig. 3. Two-dimensional analogues of σ-space, for sets of 10 data. (a) Fault data. A line from the origin represents the compatible stress state for each datum. Some have polarity (from known sense of shear); others do not. One datum is clearly an outlier, not compatible with remaining data. The rest are well clustered, their direction indicating the ratio of values of stress tensor elements. (b) Orientation and shear sense data, for the same twinnable crystal planes as detailed for (c) below. The condition $\mathbf{d.\sigma.n} > 0$ for each twinned datum (thick line) defines a half-space of compatible stress states (unflagged side of line), and a half-space of incompatible stress states (flagged side of line). The shaded sector indicates directions from the origin compatible with the twinned data. The untwinned data, shown as fine lines, give no constraint. (c) Twinnable crystal plane data, assuming a critical shear stress for twinning. For each datum, compatible stress states lie to the unflagged side of its line. The shaded polygon shows the range of stress states compatible with all data. Note the improved definition and the limit to distance from the origin, compared to (b). Data 1–4 are twinned (continuous lines) and provide a minimum magnitude of the σ-vector. Data 5–7 (dashed lines) are untwinned and limit the direction and maximum magnitude of the σ-vector. Data 8–10 (dotted lines) are untwinned but inconsequential, as their constraints lie far from any stress state compatible with the rest of the data. Note that loss of one untwinned datum (e.g. 5), or change or error in a critical untwinned datum (e.g. from 6 to 7), leads to greater loss or modification of constraint than does the loss (e.g. 4) or change (e.g. from 1 to 2) of a twinned datum.

4.4. Twinned/untwinned sets-with a critical shear stress

On the assumption of a constant critical shear stress, the hyperplane of critical conditions separating those inducing twinning from those that do not, is, for each datum in Fig. 3(c), offset from the origin, as illustrated in Fig. 1(f). To be consistent with a set of twinnable crystal planes, the σ-vector must extend from the σ-space origin to beyond the hyperplane of every twinned crystal orientation, but not as far as the hyperplane of any crystal orientation which is available for twinning but has remained untwinned. This leads to several deductions, illustrated by Fig. 3(c).

1. The locus of the σ-vector in σ-space is restricted in magnitude, not just in direction. This corresponds to restricting the absolute values of some stress parameters.
2. Both twinned and untwinned data contribute constraint to the stress state, including absolute values.
3. Twinned crystal data constrain the minimum values.
4. Untwinned crystal data constrain the maximum values.
5. Untwinned crystal data give more constraint than twinned data on the *direction* of the σ-vector, representing the *ratio* of the elements of the stress tensor, corresponding in particular to *stress ratio*.

6. The estimate of maximum stress (from untwinned planes) is less robust and less precise than the minimum stress (from twinned planes), being more sensitive to omission of critical data, to error in determination or to variability of stress within the sample.
7. A best estimate of location in σ-space for the σ-vector could be found by moving the constraining hyperplanes, while minimising a function of errors in τ_a.

The algorithm for estimating the σ-vector, mentioned in (7), above, needs to differ according to the degree of consistency of the data. In Fig. 3(c), all data are consistent with a range of σ-vector. An equivalent range in the full dimensionality of σ-space would be bounded by at least six hyperplanes. Locating a best estimate is achieved by converging the hyperplanes until only a vanishingly small locus is consistent with all data. If data are not all consistent, a few hyperplanes are diverged until a locus is obtained consistent with all data. This latter relaxation of τ_a minimises the overlap by which calculated shear stress on the most highly stressed untwinned planes exceeds that on the least stressed twinned planes. Laurent et al. (1981) summarise algorithms which are the algebraic equivalents of both these standard Linear Programming procedures.

The importance of the untwinned planes in their determinations has been well highlighted by Lacombe and Laurent (1996). Both Laurent et al. (1990) and Lacombe and Laurent (1996), after Etchecopar (1984), modify the above relaxation procedure to accord with the asymmetry between twinned and untwinned constraint (points 5 and 6, above). Their search for optimum stress tensor calculates a minimum shear stress τ_a' which is compatible with all twinned crystals while minimising the sum of all the shear stress excesses above this value of the untwinned crystals. Laurent et al. (1990) report that this method improves precision of the stress estimate.

5. Stress tensor reduction; arbitrary constraints

5.1. Scaling reduction

As illustrated in Fig. 1(a–c), Fig. 2 and Fig. 3(a), striated faults constrain the stress σ-vector in direction but not magnitude and this remains the case however many faults are taken into consideration. This accords with the contention that faults limit not the absolute values of stress parameters, but their ratios, including ratios to effective normal stress in the case of a linear friction law. So, it is conventional to impose an arbitrary scaling "reduction" on the stress tensor. This corresponds in σ-space to defining the intersection of the best σ-vector direction with a closed hypersurface, represented by the sphere in Fig. 1(a–c) and Fig. 2(a). This may be at some constant or otherwise constrained distance from the origin (Fry, 1999).

This degree of freedom, requiring scaling reduction, is absent from estimates of paleostress from deformation twinning which assume a non-zero critical component of shear stress for twinning.

5.2. Confining stress reduction

Inclusion of a friction law permits estimation of both deviatoric stress ratios and ratios to effective confining stress from a set of fault data. Otherwise, the scaling reduction of the previous paragraph still leaves not a unique best estimate of σ-vector, but a residual locus of at least one dimension permitting addition of any amount of isotropic stress. Similarly for deformation twinning, with yield stress taken to be independent of confining stress, the best σ-vector can only be constrained to an infinite locus, permitting addition of any amount of isotropic stress. Therefore, the stress tensor for both types of data is conventionally reduced to a unique specification of a deviatoric component by imposing an arbitrary condition. That used by Angelier et al. (1982), $\sigma_{11} + \sigma_{22} + \sigma_{33} = 0$, is the most common. This is equivalent in σ-space to intersecting the remaining σ-vector locus with a hyperplane through the origin (Fry, 1999).

5.3. Residual degrees of freedom

Even with the reduction of stress tensor, as discussed above, the locus of the σ-vector in σ-space may not be unique. It may extend in one or more dimensions, representing a case for which it would be inappropriate to specify a unique best estimate of stress tensor. Fry (1999) discusses and provides examples of striated fault data for which a best range of tensors is estimated. An additional procedure is proposed by Fry (1999) for additional graphical representation ("q-space"), and for the use of shear sense to limit the estimate further in such cases. This procedure is not applicable to deformation twinning. However, Laurent et al. (1990) imply that residual uncertainty is rare in analyses of calcite twinning in good samples.

6. Discussion

6.1. Assumption of common state of stress; extrinsic variation

The portrayal in the foregoing sections is deliberately simplistic, in order to draw out the essential differences of constraints on paleostress, and to enable their illustration by simple analogues. It assumes that a common stress state, tensor **σ**, existed throughout the spatial and temporal range sampled by the data. However, departures from a common stress state are inevitable. They may, in general terms, be extrinsic or intrinsic. The former are determined by boundary conditions. They accompany spatial stress gradients between boundaries of the sampled range, or changes, of the boundary conditions with time, or both.

This paper will not address *extrinsic* variation of stress state, apart from making the following general comment. The larger the distances, which separate the structures, recorded the larger the discrepancies in stress along a stress gradient. In as much as twinning provides a paleostress estimate for a single rock specimen, it is more likely to approximate a single stress state than faults sampled from a larger volume of rock.

6.2. Intrinsic fluctuation

Many comments could be made about *intrinsic* departures from a common stress state. The most important is that some fluctuation from a common state is integral to both processes considered in this paper, because both fault movements and mechanical twins are instances of localised relief of deviatoric stress by deformation. By relieving stress, they introduce heterogeneity into even an initially homogeneous stress field. By deformation, they modify the geometry of the material around them, including the location and orientation of neighbouring such structures. For real structures, a common stress state can only be an approximation.

Two generalisations follow. One is that good results are most likely to result where the structures sampled (either faults or twins) are sparse and overall deformation is slight. So, for example, one would not expect a good result from faults which are linked or curved, or have large displacements, or are repeatedly seismic. We should note that the crystallographic control of twin displacement vector, and its confining to a single crystal, make the equivalent of such worst cases impossible by deformation twinning. The other generalisation is that we may be able to learn how to interpret intrinsic departures, because they may adhere to some relationship to other inherent features, conditions or composition of our sample. Work on calcite twinning provides an example.

6.3. Calcite twinning

Can we discern some reliability in the way calcitic rocks behave when deformation dominated by twinning proceeds beyond twin initiation, to the development of abundant lamellae of visible width? Rowe and Rutter (1990) report empirical findings that both a threshold bounding shear stress required to give visibility of twins, and the eventual size and density of twins at any given stress, vary with grain size. Newman (1994) demonstrated that the distribution of grain sizes, not just a size value, should be taken into account. Their findings are in part explicable by build up of local stress concentrations at the boundaries of deforming grains of different sizes.

In the light of such reports, we should note that the algorithms of Laurent *et al.* (1990) and Lacombe and Laurent (1996), mentioned in Section 4.4 above, are mathematical and are blind to associated features of the sample. Rather than searching for a single best value of τ_a', it might be more informative to modify their algorithm to test for correlation between estimated τ_a and grain size.

However, effects at grain scale are not without their problems. Porosity, or small proportions of a different mineral, can have a marked effect on stresses at grain contacts, and may degrade possible data sets of deformation twinning in natural samples. So, despite the promise of the works cited above, all such studies are speculative. One cannot know in advance whether the data will accord with a common state of stress or not. Nevertheless, with studies of twinning, one may be able to search out the most promising sample lithologies and verify them in thin section. Generally, one cannot restrict sampling of faults to a few preferred lithologies in the same way.

6.4. Confidence in a regional common stress state

Using deformation twinning, a set of complete stress tensor estimates can be made from a sampled array of locations across a region. If the estimates are consistent, they will inspire more confidence, in either a common regional stress state or a gradational change in stress state across a region, than one or two stress tensors constructed from similarly distributed faults, for which constancy of the stress state over the region has to be assumed as part of the determination.

6.5. Fault plane isotropy

The belief that striation on a fault indicates the direction of the in-plane component of traction, and the validity of Eq. (2), assume that the fault plane is isotropic, lacking inherently preferred direction of slip. This may or may not be the case. This important assumption is not well highlighted in the literature.

7. Conclusions

1. Stress space is conceptually useful for understanding paleostress estimation.
2. At no level of constraint are deformation twin data equivalent to striated fault data.
3. A single twin datum constrains paleostress less than a single fault datum.
4. A large set of twinned and untwinned crystal data can constrain paleostress more than a large set of fault data.
5. Twinnable crystal planes, which remain untwinned, contribute most critically to the estimation of a stress tensor from deformation twinning.
6. For small sets of data, it is not possible to make general prediction as to which of the two types of data offers the greater constraint.
7. At every point in the foregoing discussion at which faults and twins are compared, it is the deformation twinning method, which is likely to give the better result or inspire

better confidence. More use should be made of deformation twin data in estimating paleostress, including corroboration of, and tests of the status of, estimates made from striated faults.

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References

Angelier, J., 1975. Sur l'analyse de mesures recueillies dans des sites faillés: l'utilité d'une confrontation entre les méthodes dynamiques et cinématiques. Comptes Rendus de l'Académie des Sciences, Paris D281, 1805–1808.

Angelier, J., Goguel, J., 1979. Sur une méthode simple de détermination des axes principaux des contraintes pour une population de failles. Comptes Rendus de l'Académie des Sciences, Paris D288, 307–310.

Angelier, J., Tarantola, A., Valette, B., Manoussis, S., 1982. Inversion of field data in fault tectonics to obtain the regional stress.—1. Single phase fault populations: a new method of computing the stress tensor. Geophysical Journal of the Royal Astronomical Society 69, 607–621.

Armijo, R., Cisternas, A., 1978. Un problème inverse en microtectonique cassante. Comptes Rendus de l'Académie des Sciences, Paris D287, 595–598.

Carey, E., Brunier, B., 1974. Analyse théorique et numérique d'un modèle mécanique élémentaires appliqué à l'étude d'une population de failles. Comptes Rendus de l'Académie des Sciences, Paris D279, 891–894.

Célérier, B., 1988. How much does slip on a reactivated fault plane constrain the stress tensor? Tectonics 7, 1257–1278.

Etchecopar, A., 1984. Etude des états de contrainte en tectonique cassante et simulation de déformation plastiques (approche mathématique). Thèse d'Etat, Université des Sciences et Techniques du Languedoc, Montpelier, France.

Etchecopar, A., Vasseur, G., Daignières, M., 1981. An inverse problem in microtectonics for the determination of stress tensors from fault striation analysis. Journal of Structural Geology 3, 51–65.

Fry, N., 1999. Striated faults: visual appreciation of their constraint on possible paleostress tensors. Journal of Structural Geology 21, 7–21.

Hobbs, B.E., Mühlhaus, H.-B., Ord, A., 1990. Instability, softening and localization of deformation. In: Knipe, R.J., Rutter, E.H. (Eds.), Deformation Mechanisms, Rheology and Tectonics. Geological Society Special Publication 54. , pp. 143–165.

Jaeger, J.C., Cook, N.G.W., 1969. Fundamentals of Rock Mechanics. Methuen, London.

Jaeger, J.C., Cook, N.G.W., 1979. Fundamentals of Rock Mechanics, 3rd edition. Chapman and Hall, London.

Jamison, W.R., Spang, J.H., 1976. Use of calcite twin lamellae to infer differential stress. Geological Society of America, Bulletin 87, 868–872.

Lacombe, O., Angelier, J., Laurent, P., 1992. Determining paleostress orientations from faults and calcite twins: a case study near the Sainte-Victoire Range (southern France). Tectonophysics 201, 141–156.

Lacombe, O., Angelier, J., Laurent, P., Bergerat, F., Tourneret, C., 1990. Joint analyses of calcite twins and fault slips as a key for deciphering polyphase tectonics: Burgundy as a case study. Tectonophysics 182, 279–300.

Lacombe, O., Laurent, P., 1992. Determination of principal stress magnitudes using calcite twins and rock mechanics data. Tectonophysics 202, 83–93.

Lacombe, O., Laurent, P., 1996. Determination of deviatoric stress tensors based on inversion of calcite twin data from experimentally deformed monophase samples: preliminary results. Tectonophysics 255, 189–202.

Laurent, P., Bernard, P., Vasseur, G., Etchecopar, A., 1981. Stress tensor determination from the study of *e* twins in calcite: a linear programming method. Tectonophysics 78, 651–660.

Laurent, P., Tourneret, C., Laborde, O., 1990. Determining deviatoric stress tensors from calcite twins: applications to monophased synthetic and natural polycrystals. Tectonics 9, 379–389.

Lisle, R.J., Vandycke, S., 1996. Separation of multiple stress events by fault striation analysis: an example from Variscan and younger structures at Ogmore, South Wales. Journal of the Geological Society, London 153, 945–953.

Malvern, L.E., 1969. Introduction to the Mechanics of a Continuous Medium. Prentice-Hall, Englewood Cliffs, N.J.

Nemcok, M., Kovác, D., Lisle, R.J., 1999. A stress inversion procedure for polyphase calcite twin and fault/slip data sets. Journal of Structural Geology 21, 597–611.

Nissen, H.U., 1964. Dynamic and kinematic analysis of deformed crinoid stems in a quartz graywacke. Journal of Geology 72, 346–360.

Newman, J., 1994. The influence of grain size and grain size distribution on methods for estimating paleostresses from twinning in carbonates. Journal of Structural Geology 16, 1589–1601.

Nye, J.F., 1957. Physical Properties of Crystals: Their Representation by Tensors and Matrices. Clarendon Press, Oxford.

Rowe, K.J., Rutter, E.H., 1990. Palaeostress estimation using calcite twinning: experimental calibration and application to nature. Journal of Structural Geology 12, 1–17.

Spang, J.H., 1972. Numerical method for dynamic analysis of calcite twin lamellae. Geological Society of America, Bulletin 83, 467–472.

Turner, F.J., 1953. Nature and dynamic interpretation of deformation lamellae in calcite of three marbles. American Journal of Science 251, 276–298.

Turner, F.J., 1962. "Compression" and "tension" axes deduced from {0112} twinning in calcite. Journal of Geophysical Research 67, 1660.

Verhoogen, J., Turner, F.J., Weiss, L.E., Wahrhaftig, C., Fyfe, W.S., 1970. The Earth. An Introduction to Physical Geology. Holt, Rinehart and Winston, Inc, New York.